Image Registration Using Hierarchical B-Splines

Zhiyong Xie and Gerald E. Farin

Abstract—Hierarchical B-splines have been widely used for shape modeling since their discovery by Forsey and Bartels. In this paper, we present an application of this concept, in the form of free-form deformation, to image registration by matching two images at increasing levels of detail. Results using MRI brain data are presented that demonstrate high degrees of matching while unnecessary distortions are avoided. We compare our results with the nonlinear ICP (Iterative Closest Point) algorithm (used for landmark-based registration) and optical flow (used for intensity-based registration).

Index Terms—Image registration, free form deformation, hierarchical B-splines, scattered data approximation, iterative closest point, optical flow.

1 MOTIVATION AND BACKGROUND

Image registration is a procedure of finding a spatial deformation to match two images (2D or 3D). It has received considerable attention in the areas of medical image analysis and computer vision [33], [24], [26], [14], [8]. For instance, in clinical applications, it can be used to match images taken from different patients or from the same patient over a period of time.

Image registration methods can be divided into landmark-based or intensity-based methods. Landmark-based methods need to first identify corresponding homologous features (points, curves, or surfaces) as landmarks. These are then mapped to each other, resulting in a deformation of the two images. Intensity-based methods try to find a deformation based on the intensity of two images directly.

In this paper, we discuss applications of some concepts from CAGD and computer graphics: free form deformation, hierarchical B-splines, scattered data approximation, on both kinds of image registration. The methods are different from the traditional B-spline registration in two ways. First, we convert different kinds of registration problems into a scattered data approximation problem. This provides an efficient solution to corresponding problem. Second, we use hierarchical B-splines for multilevel nonlinear registration. A major problem of traditional B-spline registration is the selection of the number of B-spline parameters (B-spline control points). If we only select a few parameters, only a rough match can be achieved. Using a large number of parameters, the deformation may exhibit local oscillations. The proposed methods in this paper overcome this problem since they are adaptive and put more effort in complex regions which cannot be matched by low-level deformation.

2 FREE FORM DEFORMATION WITH HIERARCHICAL B-SPLINES

Many applications in animation and shape modeling require that an object be modified on a global scale. A solution to this problem is given by free form deformations (FFD). FFD deforms an object by warping the whole space in which the object is embedded instead of warping the object directly [23], [29], [4], [10]. An analogy is changing the shape of flexible rubber; an object embedded in the rubber will be distorted.

Due to their many desirable properties such as local control, smoothness, and computational efficiency, cubic B-spline surfaces are widely used in surface modeling. In

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Manuscript received 17 July 2002; revised 7 Nov. 2002; accepted 15 Nov. 2002.

For information on obtaining reprints of this article, please send e-mail to: tvcg@computer.org, and reference IEEECS Log Number 116976.
free form deformation, they (or, rather, their volumetric generalizations to volumes) may be used by embedding an object into a B-spline volume and then changing the shape of the object by moving the B-spline control points, as Fig. 1 illustrates.

A tricubic B-spline volume has the form:

$$f(x, y, z) = \sum_{i=0}^{r} \sum_{j=0}^{s} \sum_{k=0}^{t} b_{ijk} N_i^3(x) N_j^3(y) N_k^3(z).$$

It is a $C^2$ continuous map of $\mathbb{R}^3$ to $\mathbb{R}^3$.

The $N_i^3(x)$ are cubic B-spline basis functions. They are determined by the knot spacing that splits the domain into several intervals. $b_{ijk}$ are the control points of the B-spline volume. If the knot spacing is given, the value of the function $f$ only depends on the control points. Each control point influences the hyperpatches around it. Moving a control point only changes those hyperpatches that share the control point. The scale of this change depends on the size of the hyperpatches (refer to [15, chapters 8 and 16]). This property provides a flexible way of moving the control points to adjust the local shape of the object without affecting the regions far from it.

Forsey and Bartels proposed a method for shape modeling based on this local control property. This method constructs a complex B-spline surface by manipulating the control points of the B-spline [17]. Areas where complex shapes are to be constructed can be broken into small patches by the process of knot insertion that locally refines the B-spline control polygon without changing the whole surface ([15, chapter 8]). Then, the refined control points can be manipulated to model the surface more locally. B-spline surfaces with such a hierarchy are called hierarchical B-splines.

This idea can be applied to free form deformation in the same way. Initially, the source object is embedded into a “simple” tricubic B-spline volume, simple meaning the control grid is of size $4 \times 4 \times 4$. Then, the position of control points is adjusted to deform the shape of the object. In regions where the object has a complex shape, the related control grids are locally refined and adjusted to deform the shape more locally.

This interactive approach cannot be used in image registration directly. As discussed in the previous section, image registration needs to compute the displacements of the control points based on such information as point landmarks, surfaces of two objects, or intensities of two images (2D/3D). In the following sections, we will develop different methods based on different available information to compute the displacement of control points for image registration. All these problems are converted into a scattered data approximation problem. We will also discuss how this adaptive scheme makes the system valid and efficient.

### 3 Point-Based Image Registration

In image registration, one straightforward deformation method is choosing corresponding point features (called point landmarks) first and evaluating a spatial deformation to match corresponding points.

The 2D point-based image registration problem can be formulated as follows:

**Input:** $n$ pairs of source and target points $(p_i, q_i)$, $p_i, q_i \in \mathbb{R}^2$, $i = 1, \ldots, n$.

**Output:** A $C^0$ function $f$ : $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ with $f(p_i) = q_i$, $i = 1, \ldots, n$.

This naturally breaks down into two scattered data interpolation problems: one for the pairs $(p_i, q_i^x)$ and one for the pairs $(p_i, q_i^y)$, with $q_i^x$ and $q_i^y$ being the $x$ and $y$ components of $q_i$. Different spatial interpolation functions have been proposed to solve this problem. For example, a widely used one is thin-plate splines ([5, chapter 2.2]).

A discussion for different kinds of interpolation functions on image warping is given in [27].

In practice, exact matching is often replaced by approximate matching, leading to approximation rather than interpolation. In this section, we propose a method using hierarchical B-splines to match the corresponding points from global to local. The B-spline approximation discussed here is not only used for point-based registration, but surface and intensity-based registration as well.

Using cubic B-splines, the approximation problem may be stated as a linear system of the form

$$\sum_{i=0}^{r} \sum_{j=0}^{s} b_{ijk} N_i^3(x_k) N_j^3(y_k) = q_k; \quad k = 1, \ldots, n,$$

where $x_k$ and $y_k$ are the $x$ and $y$ components of $p_k$, and $r, s$ are the number of control points in each direction.

A uniform knot spacing is used in this system, the unknowns are the control points $b_{ij}$.

To avoid undesired distortion on the image, a shape constraint can be used to obtain a “wiggle-free” transformation. Suppose a B-spline surface $s$ is used for deformation, the constraint minimizes $\frac{\partial^2 s}{\partial x \partial y}$ during the evaluation of $s$. A discrete form of this constraint is

$$\Delta^{l-1} b_{ij} = (b_{i+1,j+1} - b_{i+1,j}) - (b_{i,j+1} - b_{i,j}) = 0.$$

This constraint forces the control polygon to be close to a parallelogram mesh (refer to [15, p. 282]). Another widely used constraint is to minimize the thin plate energy functional

$$\iint \left( \frac{\partial^2 s}{\partial x^2} \right)^2 + 2 \left( \frac{\partial^2 s}{\partial x \partial y} \right)^2 + \left( \frac{\partial^2 s}{\partial y^2} \right)^2 \, dx \, dy = 0.$$

Adding the shape constraints, the new linear system is overdetermined and may be solved using least squares approximation (refer to [9, p. 248] for least squares approximation).
approximation and [13] for numerical issues of curve/surface fitting with splines). In this way, we can find a displacement of B-spline control points to best match the point landmarks. This leads to a $C^2$ mapping from source image to target image.

After the initial deformation has been found, we can check for each source point $p$ how closely it is mapped to its target $q$. If the deviation is too large, the existing deformation is changed as follows:

1. The B-spline grid is locally refined by the process of knot insertion.
2. The four control points in the refined grid which are closest to $p$ are selected and recomputed such that the transformed point $p$ gets a better match with its target.

Referring to the left panel of Fig. 2, suppose that we have a knot lattice at one level. The B-spline surface in this level is used to match every source point to its target. After the deformation, point $p$ still has a large deviation from its target. To improve the match between $p$ and its target, the B-spline grid close to point $p$ (the shaded region in the left panel) is refined. This refinement is shown in the right panel of Fig. 2. The new position of point $p$ is determined by 16 control points in its neighborhood. Referring to the right panel of Fig. 2, in the refined control grid, we can recompute the four neighboring control points of $p$ (shown in white) for a better match between $p$ and its target. These four control points are only related to the data points inside the shaded area. We only need to use the points inside the shaded area to recompute the new locations of the four control points. If other control points (shown in black) are kept untouched, adjustment of these four control points has no influence on the points outside the shaded area and do not change continuity of the B-spline surface. If more than one point has a large deviation, this strategy still can be used. With this method, only a very few new control points have to be recomputed. This procedure may be repeated to refine complex regions until a satisfactory result is achieved.

Using hierarchical B-splines, we can compute the displacement of the B-spline control points for the deformation adaptively. It reduces the computational overhead when applied to large data sets where only a few regions need more attention. It is only in these regions that more parameters are introduced, thus giving this method an adaptive level of detail control. Together with the constraints and properties of B-spline surfaces, this strategy guarantees a smooth behavior of the deformation and avoids unnecessary distortions.

Fig. 3 shows an example of image registration using our point-based method. In this example, some corresponding point landmarks on two MRI brain images are identified manually and a deformation with hierarchical B-splines is performed. The left panel is the source image. The middle panel is the target image. The right panel is the deformed source image. From this figure, we can see that the shapes of the brain and its inner substructure are adequately deformed.

Comparing with other approximation methods, Hierarchical B-spline deformation has the following favorable properties:

1. Smoothness and evenness. It is a $C^2$ map for the whole image. The deformation has global-to-local influence (we begin with a B-spline surface that has $4 \times 4$ control points, it is a global polynomial). From Fig. 3, we can see the grid is deformed evenly although the point landmarks are distributed unevenly.
2. Computational efficiency. In this example, it only takes two seconds on a 600-MHZ Pentium PC to calculate the B-spline control points.

Identifying 3D point landmarks for MRI brain images is very difficult. It may need specific software and interaction with experts. Due to the unavailability of enough point landmarks in 3D data, this method is only implemented for 2D. The theoretical extension from 2D to 3D is straightforward.
4 SURFACE-BASED IMAGE REGISTRATION

In practical applications, it may be impossible to identify enough corresponding point landmarks on two images. However, it is possible to identify some corresponding surfaces or curves on the object in two images with image segmentation and edge detection methods. If a deformation can be found to match corresponding surfaces/curves, naturally it can be applied on the whole image to match two images. This method is called surface-based image registration. The problem can be formulated as follows:

Input: a set of points of source surface 

Output: a spatial deformation \( f \) such that \( D_{surf}(f(S_1), S_2) \) is minimized. \( D_{surf}(S_1, S_2) \) is a distance measure between two surfaces \( S_1 \) and \( S_2 \).

One of the most popular methods to solve this problem is the ICP (iterative closest point) method. This method superimposes two surfaces by pulling or pushing points on the source surface to their closest point on the target surface iteratively. In this method, the distance from one point \( p \) to a surface \( S \) is defined as the distance of point \( p \) to its closest point on \( S \). \( D_{surf}(S_1, S_2) \) is defined based on the distance of all points on \( S_1 \) to surface \( S_2 \). The ICP method is first proposed to compute the best rigid or linear transformation to match two surfaces [3], [7]. Feldmar and Ayache [16] extend the algorithm to three levels of transformation: rigid, affine, and local affine transformation. Lavellee et al. [21] proposed an elastic registration method by adding an additional regularizing cost function. In this section, we will present a surface matching algorithm that extends the ICP method to multilevel nonlinear matching using hierarchical B-splines. By matching two surfaces from coarse-to-fine iteratively, we can achieve smooth and accurate matching between two surfaces with low overall distortion.1

4.1 Iterative Closest Point (ICP)

Assuming we have a source point set \( \{p_i | i = 0, \ldots, n\} \) and a target point set \( \{q_j | j = 0, \ldots, m\} \), the ICP algorithm can be summarized as follows:

1. In iteration \( k \), to each source point \( p_i^k \), find its closest point, say \( q_j^k \), in the target point set. Initially, \( p_i^0 = p_i \).
2. Evaluate the best rigid transformation \( R^k \) such that \( \sum (R^k(p_i^k) - q_j^k)^2 \) is minimized.
3. Apply the transformation \( R^k \) to each source point \( p_i^k \) to get a new point set \( p_i^{k+1} \), where \( p_i^{k+1} = R^k(p_i^k) \).
4. Terminate the iteration according to the termination criterion. Otherwise, let \( k = k + 1 \), repeat the steps from 1 to 4.

Feldmar and Ayache [16] present several possible termination criteria:

a. The distance between the two point sets is below a fixed threshold.

b. The difference between the two computed transformations at two successive iterations is below a fixed threshold.

c. A maximum number of iterations is reached.

1. While the presented method yields acceptably low levels of distortion, we do not claim to obtain the theoretically minimal distortion.

The distance transform provides an efficient way to compute the closest target point of every source point. After applying the distance transform, each feature pixel is assigned a value of zero. Each nonfeature pixel is given a value that is the distance to the nearest feature pixel. Numerous methods can be found for an approximated or exact distance transform [6], [28]. In our system, we use the algorithm proposed by Borgefors [6] due to its computational efficiency.

The ICP algorithm has been successfully applied to surface (or curve) matching if only a rigid (or affine) transformation is needed [3], [7]. When using a nonlinear deformation function in this procedure, it suffers two problems. The first one is the mismatch problem. Because the nonlinear deformation can change one shape arbitrarily, it may map the source point to the estimated target (the closest point) directly without further iteration, thus the resulting deformation depends on the initial position of the two objects. This is not desired in image registration. Another problem is the influence on the regions other than surface. Fig. 4 shows two ways to match the contours on the top left to that on the top right panel. Although the deformation on the bottom left can match these contours very well, it will distort the image when applying on image registration. The smooth deformation on the bottom right is more desirable.

To find the correct mapping between corresponding points, Feldmar and Ayache [16] use curvature and normal information as well as the spatial coordinates to define the closest point. However, the curvature and normal information only reflect the local properties of the surface. If the surfaces have numerous small ridges and valleys, the local information may lead to an incorrect match.
4.2 Surface Matching with Hierarchical B-Splines

To avoid unnecessary distortion as well as achieving a precise match, we also employ the strategy of multilevel deformation. First, rigid transformation is performed by matching the centroids and principal axes of two surfaces. This may need user interaction to make sure two surfaces have similar orientation. Next, a linear transformation is used in the ICP algorithm to remove the linear difference. Then, B-spline deformation is performed in the ICP algorithm. To overcome the problems discussed in the last section, hierarchical B-splines are employed for a smooth, global to local deformation. Beginning from a global cubic B-spline volume which has a $4 \times 4 \times 4$ control grid, we can update the position of the control points iteratively to match two surfaces until the result cannot be improved any further. If there are still some large deviations in some regions, the B-spline control grid of those regions is broken into finer control grid. This provides more degrees of freedom to modify the shape of the source surface in that region. The refined control point influences a smaller region than the old one and the modification of the shape will be localized into a smaller scale. The control points in the refined grid are adjusted to improve the matching accuracy by using the ICP algorithm. This procedure can be repeated until two surfaces achieve a satisfactory match.

The basic algorithm can be outlined as follows:

1. Let $i = 0$, the initial deformation function $f$ is a B-spline volume with a $4 \times 4 \times 4$ control grid. Embed the source surface into the B-spline volume.
2. Find the control points related to the regions where source and target surfaces have a large deviation (large distance to the closest point). These control points are called FTM (free to modify) control points.
3. Compute least squares displacement of FTM control points to match the corresponding source points and their closest points on target. This will lead to a better match between the two surfaces.
4. Check the termination criterion $a$ from Section 4.1. If it is not satisfied, go to Step 2 and repeat.
5. Check the termination criteria $a$ and $c$ from Section 4.1. If neither is satisfied, let $i = i + 1$, refine the control grid by knot insertion. Go to Step 2 and repeat. Otherwise, stop.

To compute the displacements of B-spline control points, the least squares solution of parameters is computed in each iteration such that the sum of squared distance between source point and its closest point on target is minimized. It is a linear system and can be solved efficiently (refer to Section 3 for computing the least squares B-spline control points). With this hierarchical deformation, we increase the number of deformation parameters gradually and match the feature of two surfaces from coarse to fine. Together with the properties of B-splines, we can obtain a smooth deformation for the whole image and avoid unnecessary distortion.\(^2\)

\(^2\) Other deformation functions may not have such properties. For instance, when using global polynomial deformation, we can also increase the number of parameters gradually, but it usually introduces undesirable distortions if high order polynomials are used.

Fig. 5 shows an example of this hierarchical deformation. In this example, two boundaries (inside and outside) are identified semi-automatically. The MRI brain images in the left column, from top to bottom, are the source brain, the target brain, the deformed source brain, and the hierarchical control grid. To match the source and target brains, the boundaries of the brains and their substructure are identified and matched together. From these images, we can see the deformed source image has a shape similar to the target image. The hierarchical control grid shows where it puts more effort to match the two images. The right column presents some matching results at different levels. The top image is the original location of the boundaries. The second image is the matching result after an affine transformation. The third image is the result after two level B-spline deformations. The last image is the final result (B-spline deformation is refined into six levels). The boundaries achieve a good match. From these images, it can be seen how the boundaries are matched from coarse to fine. The grid in the image also shows information indicating the smoothness of the deformation. This method is efficient in that it takes 5 seconds to match two $256 \times 256$ images on a 600-MHZ Pentium PC with 128M memory.

Fig. 6 is a 3D example. The top-left panel is the source brain. The top-right panel is the target brain. The bottom-left panel is the source brain after affine registration. The bottom-right panel is the source brain after hierarchical B-spline deformation. In this example, image segmentation is first performed to extract the brain surface. Due to the anatomical complexity of the human brain (deep ridges and valleys are distributed in different numbers and patterns between different patients), high-level deformation may force one valley to match a wrong target (the nearest feature points) if its correspondence does not exist. In this registration, B-spline volume is only refined two times (it takes 9 seconds). From these images, we can see that affine transformation only removes the linear difference between two surfaces. After the B-spline deformation, the overall shape between two brains is very similar.

5 Intensity-Based Image Registration

In many applications, we need to evaluate the deformation indirectly from the intensity of two images. The intensity-based registration problem can be formulated as follows:

**Input:** The source image $I_s$ and the target image $I_t$ are intensities of source and target images at point $p$, respectively.

**Output:** A spatial transformation $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ such that the similarity between $I_s(p)$ and $I_t(f(p))$ is optimized.

A popular way for solving this problem is first defining a similarity measure between two images and employing an optimization strategy to find the spatial deformation that optimizes the similarity. For instance, Woods proposed a method that used sum of squared differences of intensities between two images as similarity measure to compute the optimal parameters of rigid, affine, and global polynomial transformation [35]. Other widely used similarity measure are correlation [11] and mutual information [34].
For optimization-based methods, one critical issue is how to avoid local minima during the searching process. A point \( x_0 \) in the domain of a function \( f(x) \) is called a local minimum of the function \( f(x) \) in case there is a number \( a > 0 \) so that \( f(x_0) \leq f(x) \) for all \( x \) with \( \|x - x_0\| < a \). With a high-level deformation (large number of parameters), the search space is high-dimensional. Finding global minima in such spaces is a nontrivial task. Another issue is the computational efficiency. When performing a large deformation where over \( 10^6 \) parameters need to be evaluated to match the detail, the optimization method seems expensive. In this section, we present a registration method based on hierarchical B-splines, scattered data approximation, and optical flow. The proposed method can match the detail of two images with a large number of parameters (over \( 2 \times 10^6 \) parameters compared with several hundred parameters in SPM normalization [2] and AIR [35], two widely used packages). By performing multilevel deformation on a multiresolution image pyramid, this method can reduce the possibility of being trapped in a local minimum and computational overload. The implemented method is very efficient (matching two \( 256 \times 256 \times 124 \) data sets in 11 minutes on a 420-MHz Pentium PC with 256M memory, comparable to the large deformation proposed in [18] that takes several hours on a large parallel computer).

5.1 Iterative Hierarchical Registration Using Optical Flow

Optical flow is a visual displacement flow field associated with the variation in an image sequence. Working on a set
of successive images of a time-varying scene, for each pixel in one image, optical flow is used to search for the matching pixel in another image. It provides a way to estimate the displacement of each source pixel (or voxel) for a better match with the target. One of the simplest estimators is:

\[ \mathbf{v} = \frac{(I_t(p) - I_s(p))\nabla(I_t(p))}{||\nabla I_t(p)||^2} \]

where \( \mathbf{v} \) is the estimated displacement of \( p \), \( \nabla(I_t(p)) \) is the gradient of function \( I_t(p) \) at point \( p \). This estimator is based on the intensity conservation assumption (the corresponding points in source and target images have the same intensity. Refer to [30] for detail). To avoid source pixels moving too fast and skipping the matching target pixels, Thirion [32] suggested renormalizing the equation into:

\[ \mathbf{v} = \frac{(I_t(p) - I_s(p))\nabla(I_t(p))}{||\nabla I_t(p)||^2 + (I_t(p) - I_s(p))^2} \]

In practice, if two image data sets have different intensity levels, an intensity transformation should be performed [19]. Also, a Gaussian filter needs to be used to remove noise and smooth out the images ([12, pp. 42-44]).

### 5.2 Hierarchical Deformation on Multiresolution Images

One problem of the system discussed in the last section is that it does not have constraints to preserve the topology of images and assure a smooth deformation. This may be circumvented by performing a Gaussian smoothing on computed displacements. But, it cannot solve another problem which arises on the areas where \( ||\nabla I_t(p)|| \) is zero—the moving direction is not clear for points on such a region.

To solve the above problem, we use a deformation function to approximate the displacements computed at the areas that exhibit significant gradients. The idea is similar to the method discussed in Section 4. First, a rigid transformation is performed by matching the centroids and principal axes of two images [1]. Next, the least squares linear transformation is computed to smooth the estimated displacements in each iteration. If the result is not satisfactory and cannot be improved by a linear transformation, least squares B-spline deformations at different levels are computed to smooth the estimated displacements from global to local. The difference between the intensity-based and surface-based methods is that the intensity-based method uses source voxels instead of a source surface point set as source points; it uses estimated matching points instead of the closest points as targets. The region for further refinement is determined by the sum of squared differences of intensity between corresponding regions. Also, termination criteria \( a \) of Section 4.1 is changed to a threshold criterion on the difference between the two images.

One issue must be considered if high precision of registration is desirable. Medical images that have complicated anatomical variance may require a large number of B-spline control points. Solving the linear system discussed in Section 3 is also expensive. In this case, the B-spline approximation algorithm proposed in [22] can be used to compute the B-spline control points.

Corresponding to hierarchical deformation, multiresolution image representation provides another way for efficiency and accuracy. Given an image \( I_0 \), we can reduce the resolution and remove the detail of \( I_0 \) gradually to represent a image with different resolution. The new images, say \( I_1, I_2, \ldots, I_n \), where \( I_{k+1} \) has a coarser resolution than \( I_k \), form the multiresolution representation of the image \( I_0 \).

The simplest method to represent data in such hierarchical manner is the Haar wavelet [31]. It can be used to compress a \( 256 \times 256 \times 256 \) volume data into a \( 32 \times 32 \times 32 \) voxels data set and a detailed information data set. If needed, it can be recovered from a lower resolution to a higher resolution. Like other conventional multiresolution representation, this pyramid representation also suffers from the problem of translation variance [25], but this did not pose a serious problem in our experiment. In fact, multiresolution representation is widely used in various image registration systems because it can speed up computation and convergence, reduce the influence of noise, and provide the capability of coarse-to-fine searching [20], [32].

We will now exploit the duality that can be observed between the two hierarchies discussed so far: hierarchical B-splines and multiresolution image representation. Utilizing this duality, we can perform coarse deformations based on coarse data and improve the accuracy by performing finer deformations on finer data. For example, when rigid or iterative affine transformations are applied, the image detail is not crucial. So, a \( 32 \times 32 \times 32 \) resolution is sufficient to compute the deformation. To improve the accuracy, we can recover the data to \( 64 \times 64 \times 64 \) resolution.
and use B-spline volumes with a small number of control points for the deformation. This procedure progresses toward a finer level until a satisfactory result is received. Fig. 7 illustrates the basic idea of this scheme. The left side represents the number of control points used in registration. The right side represents image resolution.

This method can also provide a way to reduce memory overload. When a large data set is used in registration, the data sets at different levels can be divided into some small regions and stored. These data files can be loaded into memory as needed. When a high-level deformation is used for detail matching, most regions have been matched very well. Only some local regions need to be refined. So, we only need to load the regions which need more attention and use the data in these regions to evaluate the high level deformation. With this strategy, we can avoid using the entire data set while obtaining a satisfactory match.

5.3 Experiments

This method was implemented for both 2D and 3D image registration. Fig. 8 shows an example of a 2D hierarchical image registration of an MRI T1 image from a human brain. The top-left panel is the source data. The top-right panel is the target data. The bottom-left panel is the deformed source. Comparing the deformed source image with the target, we can see the deformed source is very similar to the target. The bottom-right image shows the displacement vectors between deformed and original source pixels.

The 3D generalization of this method was tested using 17 MRI T1 data sets of human brains (7 raw data sets and 10 data sets on which the skull is removed) from different patients. The data sets were selected randomly from those available at Good Samaritan Hospital, Phoenix. For each group of data, we arbitrarily selected assigned source-target combinations. Fig. 9 shows one example for visual checking. The three images of the top panel are three cuts of the source data at one point along three directions. The target data and deformed source data were cut at the same position and same direction. Three images on the second panel are three cuts from the target data. Three images on the third panel are cuts from the deformed source data after linear transformation. The bottom panel shows three images from the deformed source data after nonlinear transformation. After nonlinear deformation, the same detail features on both images are clearly visible.

For quantitative analysis, the correlation coefficient is used to measure the similarity between two images before and after registration. Given two images $I_s(p)$ and $I_t(p)$, the correlation coefficient can be defined as:

$$\frac{\sum_p (I_s(p) - \mu_s)(I_t(p) - \mu_t)}{\sqrt{\sum_p (I_s(p) - \mu_s)^2} \sum_p (I_t(p) - \mu_t)^2},$$

where $\mu_s$ is the mean of intensity of the template image and $\mu_t$ is the mean of intensity of the subject image. The better the two images are matched, the larger the correlation coefficient will be. If two images are identical, their correlation coefficient will be 1.

Table 1 shows the average correlation coefficients of raw data sets with their target after different levels of registration. Before registration, the correlation coefficient is 0.467. We can see the precision is improved with the increment of deformation levels.

The experiments also showed our system can converge faster and achieve a higher precision if the skull is removed. All correlation coefficients of 10 data sets are around 0.985 ± 0.003 after registration. To the raw data with skull (256 × 256 × 88 data sets), the average runtime is 18 minutes on a 420-MHZ Pentium PC with 256M memory. To data sets without skull (256 × 256 × 124 data sets), the average runtime is 11 minutes on the same machine.

6 Conclusion and Future Work

We have discussed the application of hierarchical B-splines to image registration. Three methods are proposed based on three kinds of available information. Hierarchical B-splines, in the form of free form deformation, provide a natural way
for image registration due to its global-to-local influence, coarse-to-fine matching, and computational efficiency. Since the point and surface-based methods do not use intensity information to compute the deformation, they can be used in multimodality image registration (images come from different kinds of scanners, for instance, one from PET and one from MRI). The limitation is that they depend on the availability of point/surface landmarks. If the images come from the same kind of scanner, the intensity-based method is more convenient to use. For some applications, including constraints may be very important. For example, in breast image registration, it is desirable to preserve

![Image Registration Example](image.png)

**Fig. 9.** Example of 3D intensity-based image registration. Top panel: source images. Second panel: target images at corresponding position and orientation. Third panel: corresponding source images after linear transformation. Bottom: corresponding source images after nonlinear transformation.

**TABLE 1**

Correlation Coefficients after Different Levels of Registration

<table>
<thead>
<tr>
<th>Registration levels</th>
<th>Affine</th>
<th>Level 1</th>
<th>Level 3</th>
<th>Level 5</th>
<th>Level 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of parameters</td>
<td>12</td>
<td>125</td>
<td>1,331</td>
<td>42,875</td>
<td>2,248,091</td>
</tr>
<tr>
<td>Correlation coefficient</td>
<td>0.770</td>
<td>0.858</td>
<td>0.906</td>
<td>0.933</td>
<td>0.965</td>
</tr>
</tbody>
</table>
volume during deformation. Also, extra attention needs to be paid to the consistency of registration. When registering image A to image B, we need to make sure to get the same correspondence if image B is registered to A. Our current system cannot fully guarantee this. We plan to address this problem in the future.

Validation is an important issue in image registration. Unfortunately, there is no standard method to evaluate a system. We are currently working on the validation of our intensity-based registration system using different kinds of methods with available data. More results will be reported in the near future.

ACKNOWLEDGMENTS

This work was funded in part by the Arizona Alzheimer’s Disease Research Center and Grant MH57899 from the US National Institute of Mental Health and the US National Institute on Aging and by a US National Science Foundation KDI grant to Arizona State University. The authors would like to thank Dr. Eric Reiman and Dr. K.W. Chen of Good Samaritan PET center for all their help with this research.

REFERENCES


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